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TECHNICAL ARTICLE

# Optimum Sampling Density for the Prediction of Acid Mine Drainage in an Underground Sulphide Mine

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**Abstract** Information theory allows one to define an optimal sampling density. Sampling above this critical frequency adds very little accuracy to the mapping results. We demonstrate the establishment of a critical sampling grid for the Stratonion mixed sulphide mining area in Chalkidiki peninsula, Greece, and the extraction of maximum information; the data used was derived from a previous sampling campaign carried out to estimate ore reserves and predict the net neutralization potential (NNP) of the rock formations. A structural analysis of NNP values generates a variogram model that can be used to define the optimum sampling grid.

## Introduction

The prediction of acid generation potential in geological formations enables the implementation of an environmentally sound exploitation strategy and closure plan (Adam et al. 1997; Komnitsas et al. 1998). Such predictions are normally made based on static and/or kinetic evaluations of samples from the mine site; in most cases, several experimental studies are required due to the mineralogical complexity of the materials involved. Acid–base accounting (ABA) is a widely applied static test that determines the balance between acid generating and acid consuming phases (Sobek et al. 1978). It involves estimating (1) the acid potential (AP) by considering oxidation of the total sulphur, (2) the neutralization potential (NP), and (3) the net neutralization potential (NNP) by subtracting NP from AP. The values of AP, NP, and NNP are expressed in kg $CaCO_3$/t material. The accuracy of this test is sometimes criticised, since some sulphur may be present in forms that are not amenable to oxidation. If the mineralogy of the formation varies widely, samples must be examined and assumptions must be made to predict the overall acid generation potential.

Modelling and geostatistical techniques can be also used to assess the characteristics and provide a reliable prediction of the acid generation potential in geological formations or waste disposal sites (Komnitsas and Modis 2006; Modis and Papaodysseus 2006; Modis et al. 1998, 2007). The accuracy of geostatistical and other estimators depends on the quality of the sampling campaign. An important question that should always be answered, regardless of the estimation procedure, is how representative the samples are. To determine the optimum grid size, the usual practice is to employ estimation variance as a criterion of efficiency (David 1976; Dowd and Milton 1987); various grid setups are considered and the average estimation variance is plotted as a function of sampling density. The optimal grid size is the threshold beyond which no further improvement of the estimation variance is seen.

Past mining and waste disposal activities as well as the continuous generation of AMD have affected the ecological quality of the Chalkidiki peninsula (Gaidajis 2003; Kelepertsis et al. 2006; Lazaridou et al. 2004; Nikolaidou 1998). Therefore, the prediction of AMD potential of all materials involved is very important to enable future sustainable mining and minimize environmental impacts. We

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used a mathematical approach based on information theory to estimate the optimum sampling density and predict AMD generation at the Madem Lakkos and Mavres Petres mines. This approach enables the establishment of a theoretical optimum sampling grid size with a minimum mean square error, and overcomes the limitations of the subjective and awkward process of iteratively calculating kriging estimation error for various sampling grids by progressively increasing the sampling density until some improvement is seen.

## Mining history: geology of the Stratonion area

Kassandra lies in the Chalkidiki prefecture, about 100 km from Thessaloniki, in northern Greece, and has been the largest mixed sulphide mine site in the country. Ancient mining reached a peak in the area during the time of Phillip II and Alexander the Great, when silver and gold financed their conquests of the then-known world between 350 and 300 BC. The lead-rich ores from the Madem Lakkos mine at Stratonion were smelted for silver and the Olympias arsenopyrite ores were processed for their high gold content. It has been estimated, from the volume of ancient slags, that about 1 million tonnes (t) of ore was extracted from each locality during that period. It is believed that by 300 BC, the bulk of the ores above the water table at Olympias had been exploited, though the Stratonion mine continued production through the Roman, Byzantine and Turkish periods. Ancient mining is less well documented at Skouries, but the presence of abundant slags and the name itself (Skouries means slags) provide evidence of ancient smelting activity (ACA Howe International Ltd. 2004).

The Stratonion mines of Mavres Petres and Madem Lakkos lie some 6 km to the south, 3–3.5 km W-NW of the port and loading facility in the village of Stratonion (latitude: 40°31′0N, longitude: 23°49′60E). The Olympias mine is situated some 2.3 km W of the coastal town of Olympias and 22 km N of Stratonion. The Skouries site is situated about 11 km SW of the Stratonion mines, 11 km S of the town of Paleohori, and 3 km NE of the village of Megali Panagia.

A number of companies have operated in the area over the years. Hellenic Chemical Products and Fertilisers Co. (HCP&FC) operated until 1992, followed by TVX Hellas, owned later by Kinross, which in May 2003 filed for bankruptcy and closed its mining operations. The mineworkers protested against the closure and in December 2003, the Greek government announced a series of measures to compensate and assist the redundant workers and to reopen the mines. After Kinross decided to disengage from Greece, the property was sold, subject to parliamentary approval, to the Greek consortium Hellas Gold SA. In November 2004, European Goldfields Ltd. acquired a 65% interest in Hellas Gold; Aktor S.A., which was a leading Greek construction company, had the remaining 35%. These assets in northern Greece included three near-production deposits with a 70-year concession covering an area of 317 $km^2$. The properties include the polymetallic deposits of Olympias and Stratonion, which contain gold, lead, silver and zinc, and the copper–gold porphyry deposit referred to as Skouries. Estimated proven reserves at Olympias were 10 Mt of ore grading 8.1 g/t gold, 108 g/t silver, 3.5% lead and 4.6% zinc. Estimated probable reserves were 4 Mt grading 9.7 g/t gold, 148 g/t silver, 4.9% lead, and 6.6% zinc. Estimated proven reserves at Stratonion were 1 Mt grading 191 g/t silver, 8% lead, and 10% zinc. Estimated probable reserves were 862,000 t grading 189 g/t silver, 8% lead and 11.7% zinc. The Skouries gold–copper porphyry deposit probable reserves were estimated to be 129 Mt at grades of 0.89 g/t gold and 0.56% copper (Newman 2004).

The origin of the Madem Lakkos and Olympias ores is subject to controversy. Nikolaou and Kokonis (1980) supported a metasomatic replacement origin for the ores at the Olympias deposit based on geological evidence and ore textures. Nebel et al. (1992) suggested a pre-metamorphic syngenetic origin for the Olympias and Madem Lakkos ores, which were subsequently regionally metamorphosed and later reworked by Tertiary hydrothermal activity related to igneous rocks in the area. On the basis of geochronological data, Gilg and Frei (1994) favoured an epigenetic origin for the ores and a genetic relationship to the emplacement of Tertiary porphyritic stocks in the area of eastern Chalkidiki. Kilias et al. (1996) also studied the microthermometry of synore gangue material from undeformed and deformed Olympias ores.

The orebodies are mainly developed in the marbles or in the contacts between the marbles and other formations. The form of the deposits is often irregular; however, it is controlled by the nature of the contact between the marbles and the other rocks.

The Mavres Petres ore body is a hydrothermal to mesothermal complex sulphide deposit situated in a relatively small thickness marble, or at the contact between the marble and the surrounding schists or gneisses. The Madem Lakkos ore body is also a hydrothermal complex sulphide deposit, situated mainly at the upper contact of a lenticular body of marble of considerable thickness with the overlying gneiss. A typical cross section of the Madem Lakkos sulphide deposit is shown in Fig. 1.

The ore in the Stratonion mines was mined underground by the cut-and-fill method. Earlier, sub-level caving was employed and this resulted in the generation of extensive cracks in the overlying strata, facilitating water infiltration and subsequently acid generation.

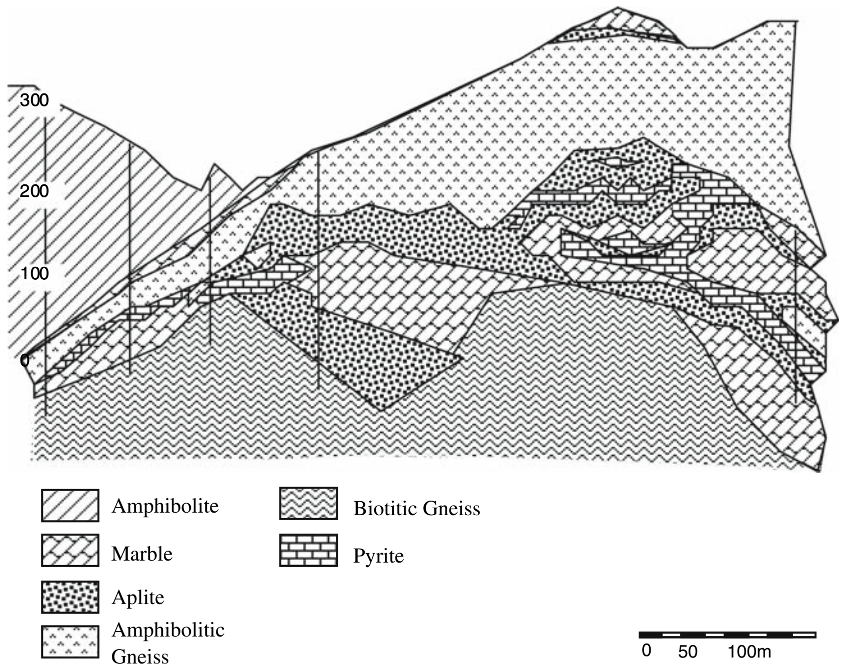


**Fig. 1** A typical cross section of the Madem Lakkos sulphide deposit (courtesy Hellenic Chemical Products and Fertilisers Company)

## Related notions and theorems from sampling theory

Originally developed for deterministic electrical signals, information theory deals with an estimation problem when a signal has to be reconstructed from its samples. Reconstruction involves filtering of the sampled signal by an interpolation filter; if the conditions of the sampling theorem are satisfied, then the original signal is exactly reproduced (Papoulis and Pillai 2002).

In a band limited function, the spectrum vanishes above a certain frequency limit. The spectrum of an arbitrary sampled band limited function is a scaled, periodic replication of the spectrum of the original function (Pratt 1991). If there is no spectrum overlapping, namely if the sampling frequency is greater than a critical value called the Nyquist rate, then the original function can be reconstructed from its samples by linear filtering. The transfer function of the filter must be such that when applied to the samples sequence spectrum, the spectrum of the resulting signal is identical to the original. Thus, the filter is used to clear the spectrum from all replicas and keep only one. Based on this criteria, in this application, the only appropriate transfer function of the interpolation filter is the square pulse.

A product in the frequency domain is a convolution in the time or space domain. The sin $c$ function

$$\sin c(x) = \begin{cases} 1, x = 0 \\ \dfrac{\sin(\pi x)}{\pi x}, x \neq 0 \end{cases} \tag{1}$$

is the continuous inverse Fourier transform of the rectangular pulse $\Pi(x)$ of width $2\pi$ and height 1:

$$F[\Pi(x)] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\omega x}\,\mathrm{d}x = \sin c(x). \tag{2}$$

Since the Fourier transform of the sin $c$ function is the square pulse function, the convolution of the sin $c$ function with the samples produces the original function, as shown in Fig. 2a. Thus, the sin $c$ function is the ideal interpolation filter. The critical Nyquist frequency for band limited signals is two times higher than the signal bandwidth.

Besides the square pulse function, other easier to realize in practice functions can be used as approximate transfer functions to interpolate the sampled function in one or two dimensions. All deterministic interpolation methods can be defined by selecting appropriate transfer functions (Fig. 2b, c). When the sampling rate is lower than the Nyquist limit, then overlapping of the infinite copies of the initial spectrum is seen and, as a result, all aforementioned approximations are intrinsically problematic. Therefore, causal approaches must be abandoned and stochastic approximations must be considered instead (Modis et al. 2007).

When information theory is applied to earth related sciences, natural variables under investigation are usually modelled as random fields. In that case, the most common random field models, described by a covariance function with an influence range "$a$", are approximately band limited (e.g. the "spherical scheme" in Fig. 3), with a sampling interval ($\Delta s$), or lag distance, defined in Eq. 3 (Modis and Papaodysseus 2006):

$$\Delta s = a/2. \tag{3}$$

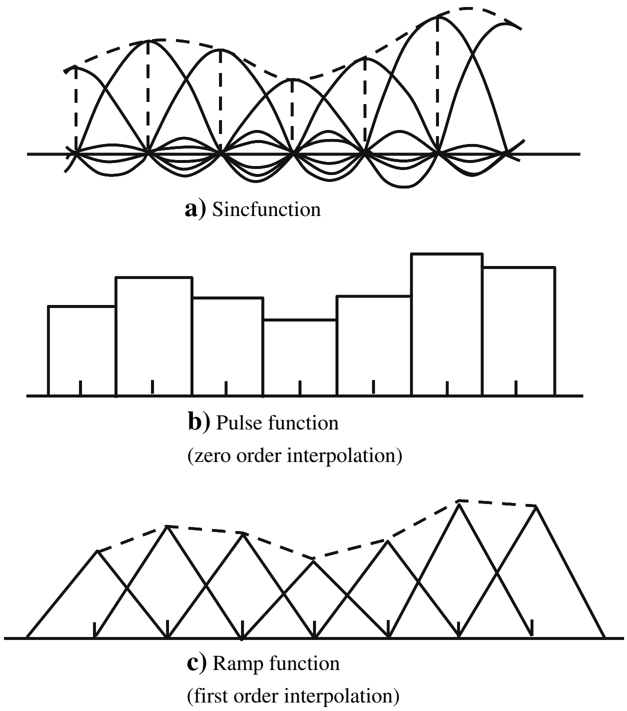


**Fig. 2** Signal reconstruction (Niblack 1986)

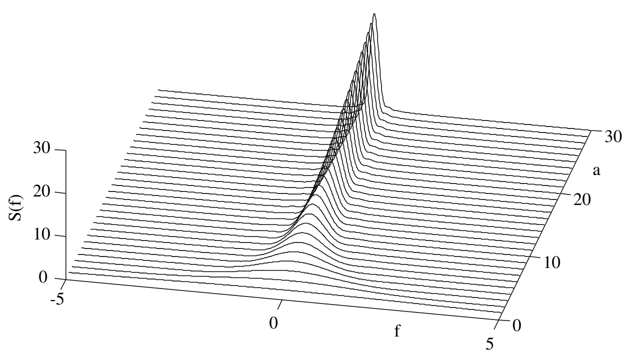


**Fig. 3** Fourier transform of the spherical correlation model as a function of range of influence "*a*" (Modis and Papaodysseus 2006)

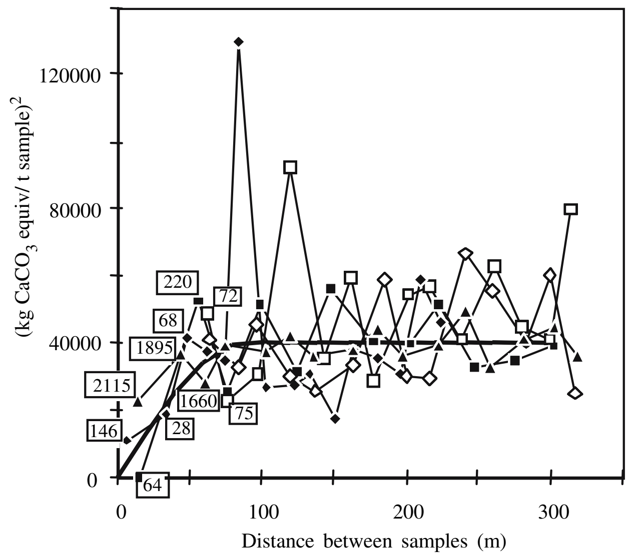


**Fig. 4** Experimental mean variograms and a fitted isotropic model for sample pairs; DTH refers to "down the hole"

In other words, if the structural analysis of an ore body reveals an underlying structure with a certain range of influence, then the optimum exploratory grid size is defined as half the value of this range.

## Results and discussion

### NNP model estimation

The NNP model estimation was based on a set of 171 exploratory drill-holes, with a depth of up to 900 m, with multiple samples collected from each drill-hole, totalling 1,820 samples covering the Madem Lakkos and Mavres Petres mining area; see Modis et al. (1998) for more details. Only 89 samples of interest, the mineralized samples, were chemically analyzed. Direct measurements of the NNP value for all 1,820 samples were not carried out due to time and budget limitations. Mean values for the different formations were used to correlate the drill-hole logs. The analytical approach followed is described in a previous publication (Modis et al. 1998).

According to geostatistics, the structural analysis of the NNP spatial distribution includes estimation of the NNP variogram function as a measure of the variability in different directions in space (Journel and Huijbregts 1978). The mean experimental variograms were calculated for five spatial directions (see Fig. 4), using sample pairs to calculate particular points. No significant difference was observed in any specific direction, so isotropy can be reasonably assumed. Therefore, the fitted function, $\gamma(r)$, is an isotropic spherical model with sill $C_o$ = 40,000 (kg $CaCO_3$ equiv/t sample)$^2$, and a range of influence, $a$ = 90 m. The model was validated using the back estimation technique: a variogram model was used to estimate each data value, which was in turn removed. The best model is the one that has the minimum average estimation error.

### Establishment of the optimum critical sampling grid

The location of each drill hole is presented in Fig. 5. It can be easily seen from this figure that not only did the

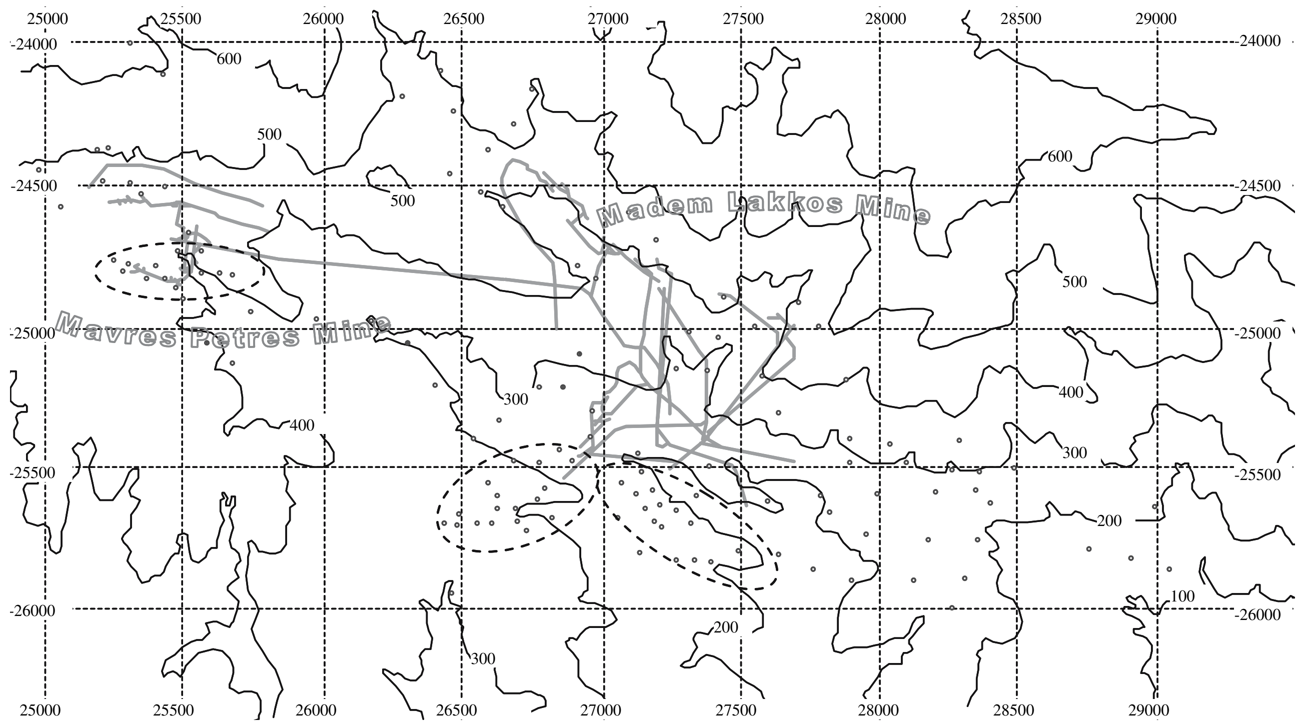


**Fig. 5** Drill-hole location and over sampled areas (seen in *ellipses*) in the Stratonion mining area

drilling not follow any grid configuration but that also the distance between the drill-holes varied widely. The closest distance between two successive drill-holes was 20 m, while the longest was 250 m; the average distance was about 150 m. A higher concentration of drill-holes is seen in the south-central (Madem Lakkos mine) and northwest (Mavres Petres mine) parts of the wider mining area. The distance between drill-holes in these areas ranges between 20 and 120 m, while in the other parts, it ranges between 100 and 250 m. Thus, a separate distance distribution with different averages could be identified for each part.

If the range of influence as defined by the structural analysis of the NNP values (Fig. 4) is inserted into Eq. (3), the size of the ideal square sampling grid becomes 45 m. However, the drill-holes are distributed unevenly, indicating over-sampling in some parts and under-sampling in others (Fig. 5). The southwestern part of the Madem Lakkos mine and the southern part of the Mavres Petres mine were over-sampled, since the drill-hole distance was as short as 20 m, which is shorter than the recommended distance of 45 m. The rest of the mining area was under-sampled, since no drill-holes are close to the optimum distance. As previously explained, this may cause problems during the process of modelling the NNP and subsequently affect the accuracy of the prediction of AMD generation of the formations under study.

## Conclusions

In general, the design of the drilling campaign can be considered as an optimization problem. The selection of an appropriate drill-hole network relative to the grid size can maximize information regarding the distribution of NNP values required for estimation and prevention of AMD and result in considerable savings in money and time. In order to increase the probability of accurately estimating the acid generation potential of a geological formation using NNP or other values deriving from implementation of standard static or kinetic tests, the optimum sampling density can be modified by taking into account the topography and geology of the site (e.g. faults, lenses of different materials, permeability of the overlying formations), as well as the mineralogy of the involved formations and the potential presence of hot spots. For example, the accuracy of the calculations could be improved if only the percentage of sulphur that is amenable to oxidation were considered.

If the density of the sampling grid is close to or greater than the critical value, the NNP numerical model can be performed using simpler interpolation algorithms, such as the inverse distance square, and the accuracy would be equal to that derived by geostatistics. However, if the available data are less than the critical sampling density, geostatistical analysis would be more appropriate.

In this study, a variogram model generated by structural analysis of NNP spatial distribution in the mining area of Stratonion, in the Chalkidiki peninsula, Greece, was used to establish a critical sampling grid; in some parts of the ore body, the sampling grid was denser than required.

The methodology presented in this paper can also be applied to identify a critical sampling grid for: (1) prediction of AMD potential in waste disposal sites, especially if different layers of wastes have been placed there over the years, due to the varying quality of the raw material and improvements in the beneficiation techniques applied; (2) assessment of soil contamination in the vicinity of mining and waste disposal sites; and (3) other environmental applications involving earth-related space-distributed data. Finally, it should be mentioned that this approach may offer substantial cost savings if the area under study is large.

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## References

ACA Howe International Ltd (2004) Technical Review of the Kassandra mines property, Chalkidiki prefecture, Greece. Berkhamsted, Herts, 15 May 2004, UK

Adam K, Kourtis A, Gazea B, Kontopoulos A (1997) Evaluation of static tests used to predict the potential for acid drainage generation at sulphide mines. T I Min Metall A 106:A1–A8

David M (1976) What happens if?—Some remarks on useful geostatistical concepts in the design of sampling patterns. In: Proceedings of the symposium on sampling practices in the minerals industry, Australasian Institute of Mining and Metallurgy, pp 1– 15

Dowd PA, Milton DW (1987) Geostatistical estimation of a section of the perseverance nickel deposit. In: Matheron G, Armstrong M (eds) Geostatistical case studies. D Reidel, Norwell, pp 39–67

Gaidajis G (2003) Ambient concentrations of total suspended particulate matter and its elemental constituents at the wider area of the mining facilities of TVX Hellas in Chalkidiki, Greece. J Environ Sci Heal A 38(11):2509–2520

Gilg HA, Frei R (1994) Chronology of magmatism and mineralization in the Kassandra mining area, Greece. The potentials and limitations of dating hydrothermal illites. Geochim Cosmochim Acta 58:2107–2122

Journel AG, Huijbregts CJ (1978) Mining geostatistics. Academic, London, p 600

Kelepertsis A, Argyraki A, Alexakis D (2006) Multivariate statistics and spatial interpretation of geochemical data for assessing soil contamination by potentially toxic elements in the mining area of Stratonion, north Greece. Geochem-Explor Env A 6(4):349–355

Kilias SP, Kalogeropoulos SI, Konnerup-Madsen J (1996) Fluid inclusion evidence for the physicochemical conditions of sulfide deposition in the Olympias carbonate-hosted Pb-Zn (Au,Ag) sulfide ore deposit, E. Chalkidiki peninsula, N. Greece. Mineralogica Deposita 31:394–406

Komnitsas K, Kontopoulos A, Lazar I, Cambridge M (1998) Risk assessment and proposed remedial actions in coastal tailings disposal sites in Romania. Miner Eng 11:1179–1190

Komnitsas K, Modis K (2006) Soil risk assessment of As and Zn contamination in a coal mining region using geostatistics. Sci Total Environ 371:190–196

Lazaridou-Dimitriadou M, Koukoumides C, Lekka E, Gaidagis G (2004) Integrative evaluation of the ecological quality of metalliferous streams (Chalkidiki, Macedonia, Hellas). Environ Monit Assess 91:59–86

Modis K, Adam K, Panagopoulos K, Kontopoulos A (1998) Development and validation of a geostatistical model for prediction of acid mine drainage in underground sulphide mines. T I Min Metall A 107:102–107

Modis K, Papaodysseus K (2006) Theoretical estimation of the critical sampling size for homogeneous ore bodies with small nugget effect. Math Geol 38(4):489–501

Modis K, Papantonopoulos G, Komnitsas K, Papaodysseus K (2007) Mapping optimization based on sampling size in earth related and environmental phenomena. Stoch Env Res Risk A. doi: 10.1007/s00477-006-0096-8

Nebel ML, Hutchinson RW, Zartman RE (1992) Metamorphism and polygenesis of the Madem Lakos polymetallic sulfide deposit, Chalkidiki, Greece—a reply. Econ Geol 87:1187–1190

Newman HR (2004) The mineral industry of Greece. US Geological Survey Minerals Year book, pp 13.1–13.8

Niblack W (1986) An Introduction to digital image processing. Prentice-Hall, New Jersey, p 215

Nikolaidou AE (1998) Selenium content in sulphide ores from the Chalkidiki peninsula, Greece. J Environ Pathol Toxicol 17(3–4):189–197

Nikolaou M, Kokonis I (1980) Geology and development of the Olympias mine, eastern Chalkidiki, Macedonia, Greece. In: Jones MJ (ed) Complex sulphide ores. Institute Mining Metallurgy, London, pp 260–270

Papoulis A, Pillai U (2002) Probability, random variables and stochastic processes. McGraw Hill, New York, p 852

Pratt WK (1991) Digital image processing. Wiley-Interscience, New York, p 698

Sobek AA, Shuller WA, Freeman JR, Smith RM (1978) Field and laboratory methods applicable to overburdens and mine soils. EPA 600/2–78-054, U.S. Environmental Protection Agency, Cincinnati, Ohio, USA, p 203